Improvements to the Conservation Properties of FE-Based Models

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Document Number N0001402WX20834

LONG TERM GOAL

The ultimate goal of this work is to upgrade the forecast capability of finite element-based (FE) coastal circulation models by implementing improvements that result in the local conservation of modeled quantities, a long-term deficiency of such models in certain environmental regimes.

OBJECTIVE

The objectives of this work are to:

1) Assess the local mass conservation problem associated with FE coastal circulation models. A quantitative measure of the local mass conservation must be defined in the context of the current formulation of FE coastal models; subsequently, consideration will be given to identifying the spatial scales and dynamical scenarios in which the lack of local conservation adversely affects computed circulation.

2) Evaluate the performance of reformulated versions of two widely used FE coastal circulation models using numerical discretization techniques that improve mass conservation properties of the computed solution. Schemes under consideration include the Discontinuous Galerkin (DCG) method or a Finite Volume (FV) approach.

3) Develop metrics and tools to identify local mass conservation problems within a computed solution and evaluate numerical schemes implemented to resolve the local conservation problem. Consider computational costs, stability, and ease of implementation in addition to local conservation skill.

APPROACH

Finite element circulation models with their inherent grid flexibility in representing complex geometry and steep gradients offer a significant advantage in representing nearshore and coastal processes. These models presently utilize a Generalized Wave Continuity Equation in place of the primitive formulation for the mass conservation equation (Kolar and Westerink, 2000). This substitution eliminates folding of the dispersion relation and the resulting spurious short wave noise that results from a FE implementation of the primitive equation. A major drawback to this approach is the removal of the strict constraint on mass conservation via the continuity equation. For many shelf-scale tidal and storm surge applications mass conservation is not an issue and accurate, robust solutions are obtained. However, as coastal models are pushed to smaller scales and applied in dynamical regimes where advection is a dominant process, the lack of mass conservation has become more problematic for finite element model calculations. In

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **30 SEP 2002**	2. REPORT TYPE	3. DATES COVERED **00-00-2002 to 00-00-2002**

4. TITLE AND SUBTITLE **Improvements to the Conservation Properties of FE-Based Models**	5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Naval Research Laboratory, Ocean Dynamics and Prediction Branch,,Stennis Space Center,,MS, 39529**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S) 11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT
The ultimate goal of this work is to upgrade the forecast capability of finite element-based (FE) coastal circulation models by implementing improvements that result in the local conservation of modeled quantities, a long-term deficiency of such models in certain environmental regimes.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**7**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

particular, mass imbalances impact the accurate coupling of the hydrodynamics with transport equations for salinity, temperature and/or other conservative species.

Two numerical discretization approaches are promising for the restoration of mass conserving properties into finite element model solutions. The first is the Discontinuous Galerkin (DCG) method that has the desirable property of requiring mass conservation locally over each computational element. The second is a Finite Volume (FV) approach that by nature of its flux-based calculations is mass conserving. Some disadvantages of these methods are the increased computational cost associated with the DCG method and the first order accuracy of FV methods. Each method also has aspects of its implementation that are subject to variation and whose performance is largely untested for coastal circulation problems, e.g. the use and type of slope-limiters and/or the form of the normal (element-edge) flux calculation.

The two FE circulation models under study are those that are used both within the modeling community and in Navy-related research and operations. The first model, the Dartmouth Circulation Model (QUODDY) (Lynch et al., 1996), is the more dynamically advanced FE model of the two. QUODDY is a 3-D, fully nonlinear model that includes tidal, wind-driven, and baroclinic physics, and incorporates advanced turbulence closure. This model is typically applied over seasonal and synoptic event time scales for the purpose of developing realistic circulation climatology, assessing individual forcing contributions to overall circulation, and forecasting in limited short-term scenarios. The second model, the Advanced Circulation Model (ADCIRC) (Luettich et al., 1992), is also a 3-D fully nonlinear model that has a successful history of simulating 2D tidal, storm surge, and wave-driven circulation and allows for significant wetting/drying events; it is presently advancing to include a prognostic baroclinic component. An older 2D version of this model is currently exercised operationally at NAVOCEANO as a nonlinear simulator of tidal and wind-driven circulation. ADCIRC is the more probable candidate for real-time predictions and subsequent transition to operations, in the context of Naval applications, though it presently lags QUODDY with regard to the inclusion of baroclinic dynamics. Both finite element models are designed with modular dynamics in which certain mechanisms, such as heat flux, wind forcing, stratification, tides, or river inflow, can be independently included or excluded from model equations. This modularity in the equations and forcing readily allows the incorporation of numerical modifications to model equations and permits easy configuration of various dynamical scenarios.

The approach centers initially on the development and testing of DCG implementations within the QUODDY and ADCIRC models. A DCG form of the primitive continuity equation will replace the GWCE with the momentum and transport equations retaining their continuous Galerkin formulation. Evaluations of these models will indicate if a full DCG implementation, i.e. additional DCG implementations for the momentum and transport equations, is necessary.

A series of field tests are constructed that span the spectrum of spatial scales and dynamical forcing relevant to the coastal ocean. Specifically, such test cases include the simulation of circulation driven by tides, wind, waves, rivers and density gradients from meters to kilometers over beaches, bays, shelves and semi-enclosed seas. For each test case the extent of the mass conservation problem will be assessed using the current GWCE forms of each model.

Comparisons of DCG based solutions to solutions obtained using the GWCE version of the same model will provide a basis for validation and evaluation of conservation properties. The performance and stability of the DCG formulation itself will be addressed by 1) examining the form of the flux computation and its effect on performance of the method, 2) investigating the definition and impact of

slope-limiters vs. flux-limiters, and 3) determining the matrix condition number (a measure of stability) for different dynamical and geographical settings. Experiments will also be designed to measure the sensitivity of the DCG implementations to time step, mesh spacing, and modeled dynamics. Ultimately a performance matrix will be constructed that spans the range of dynamical forcing and spatial scales relevant to the coastal ocean. The performance metric will be based on the comparisons to GWCE based solutions, the degree to which mass is globally and locally conserved, solution accuracy, stability, computational cost, and ease of use.

WORK COMPLETED

The GWCE form of the continuity equation has been replaced with a DCG formulation of the primitive continuity equation in each of the Navy's predominant, finite element-based coastal circulation models, QUODDY and ADCIRC. This work has been achieved through close collaboration with M. J. Guillot (U. New Orleans) and C. Dawson (U. Texas), respectively. The DCG form of QUODDY is validated using the standard Gulf of Maine test case (Naimie et al, 1994) for tidal dynamics. Comparisons between GWCE and DCG computed elevations and velocities are negligible.

Within QUODDY, a node-based DCG formulation is compared to the traditional flux-based form. Note also the current DCG implementation solves a Riemann problem for the element edge normal fluxes and uses a Cockburn and Shu slope limiter; the impact of these choices on performance of the method will be investigated further.

The DCG form of ADCIRC is currently being applied in a similar manner to the Gulf of Maine application.

A series of additional test cases are defined that span a range of spatial scales and dynamical forcings. Table 1 provides the names and relevant details for all test problems.

Table 1. Definitions of 8 test problems that range in scale from 10,000 m to 2 m covering semi-enclosed seas, shelf regions, inlets, bays, and beaches. Various combinations of dynamical forcing from tides, winds, rivers, waves and density gradients are included.

TEST PROBLEM	REFERENCE	SPATIAL SCALES	DYNAMICAL FORCING
Gulf of Maine	Naimie et al, 1994	10,000 m	tides
Persian Gulf	Blain, 2000	6000 m	tides, density
Mississippi Sound	Blain and Edwards, 2002	500 m	tides, wind
Bay St. Louis	Blain and Veeramony, 2002	100 m	tides, river
Idealized Inlet	Veeramony and Blain, 2002	125 m	tides, river, wind
Idealized Inlet	Cobb and Blain, 2002a	125 m	tides, wave stress
Plane Sloping Beach	Blain and Cob, 2002	5 m	wave stress
Barred Beach w/ Channel	Cobb and Blain, 2002b	2 m	wave stress

RESULTS

The Gulf of Maine forced by periodic tidal elevations at all boundaries is the standard test problem for the QUODDY model and is used to validate the DCG computed solution as compared to the original

GWCE formulation. Tidal boundary conditions are ramped over 3 days and computations extend a total of 10 tidal cycles. Elevation (Figure 1, left) and velocities (Figure 1, right) after 3 days are computed using both the DCG and GWCE methods. Differences between the elevation, and the u and v components of velocity generally agree within 1-2% of the GWCE-based calculations.

The explicit formulation of the DCG approach implemented requires that Courant conditions remain less than unity throughout the domain which leads to a more restrictive time step than is required by the semi-implicit GWCE formulation presently in QUODDY (i.e. $1/10^{th}$ the size for the Gulf of Maine problem). The result is clearly an increase in computational times. Furthermore the stability of the DCG method cannot be ensured without some type of slope limiting incorporated into the algorithm at the element edges.

Two methods for the degrees of freedom per element were investigated. In the first, the element degrees of freedom were the element average and gradient, and in the second, they were the values at the element vertices (nodes). Each method produced similar results, although, in the second method implementation of the elevation boundary conditions was facilitated. Additionally, the element vertices method produced a simpler mass matrix that reduced the computational effort. While no stability advantages are evident, the node-based implementation enhances usability since all existing node-based pre- and post- processing software can be utilized.

IMPACT/APPLICATION

Accurate predictions of coastal circulation are essential for simulating the transport of temperature, salinity and/or other conservative species throughout coastal waters. The transport of constituents translates, for example, to the movement of sediment, the dispersion of chemicals, or the location of density fronts, all of which impact Naval and civilian operations in coastal waters. Finite element-based hydrodynamic models which are well-suited for application in the geometrically and dynamically complex regions of the coastal ocean suffer from mass imbalances at small spatial scales in highly advective flows. The continuous Galerkin finite element method presently used in their discrete representation does not rigorously impose mass conservation over local elements. Additionally the use of the Generalized Wave Continuity Equation in place of the primitive form of the continuity equation, an approach taken to eliminate sub-grid scale noise in FE models, further exacerbates the lack of local conservation in certain applications. The development here of DCG forms of these coastal circulation models will eliminate this lack of conservation (conservation over each element is required) and restore FE models as a superior tool for accurately predicting circulation and transport in coastal waters.

TRANSITIONS

Transitions at these early stages of development and testing the new DCG forms of the QUODDY and ADCIRC models are limited to sharing knowledge of implementation, and experiences of model performance with model developers, i.e. for ADCIRC: Dawson (UT), Luettich (UNC), Westerink (Notre Dame) and for QUODDY: Naimie and Lynch (Dartmouth College).

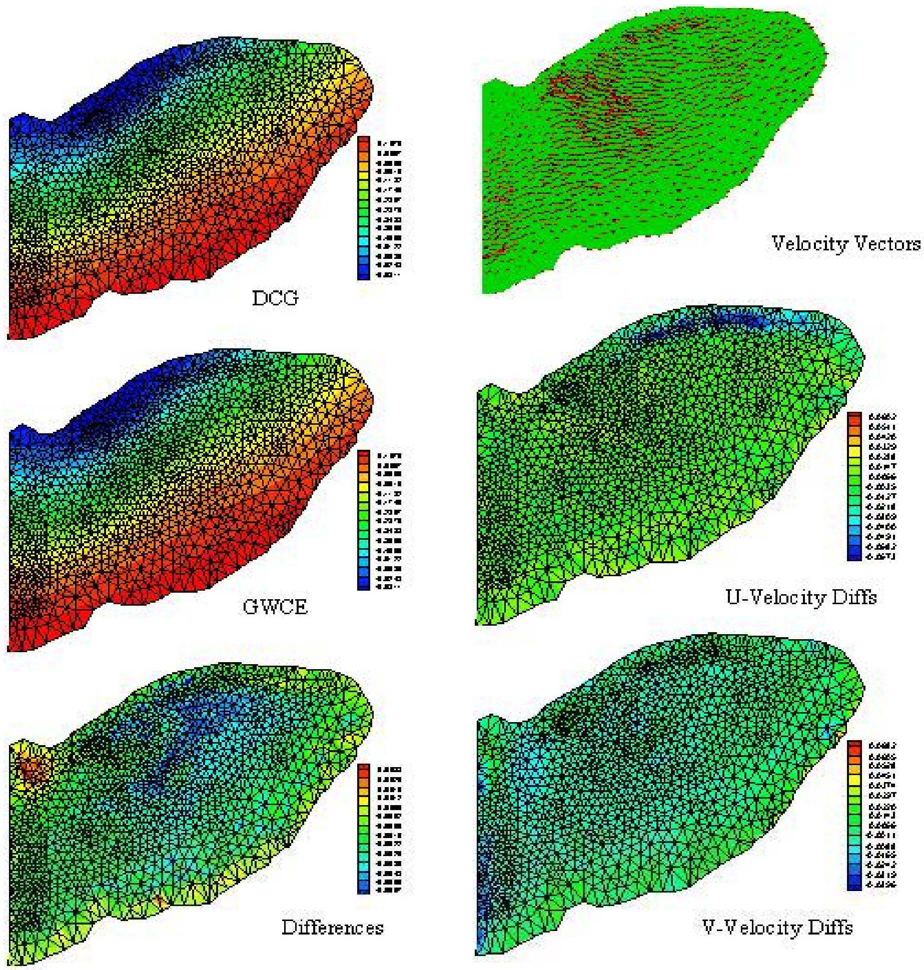

Figure 1. Comparisons between elevations (left) and velocity (right) for DCG and GWCE formulations of the QUODDY model. Errors generally within 1-2%.

RELATED PROJECTS

An active collaboration is ongoing with M. J. Guilliot (U. New Orleans) with regard to the implementation of a DCG form of the QUODDY model. Very fruitful interactions continue with C. N. Dawson (U. Texas), the first to develop a DCG form of the ADCIRC model and J. Westerink (Notre Dame) who is investigating the spatial scales and dynamics required for a switchover from the CG to DCG forms of ADCIRC. Strong interactions exist with C. E. Naimie and D. R. Lynch (Dartmouth College) regarding model development and application of QUODDY and R. A. Luettich (U. North Carolina) regarding advances in the ADCIRC model. Interactions continue with R. L. Kolar (U. Oklahoma) on the baroclinic development of ADCIRC and the use of the DCG ADCIRC model.

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